B-math 2nd year Mid Term (supplementary)
Subject : Analysis III

Time : 2.00 hours Max.Marks 40.

1. Evaluate the line integral $\int_C (x^2 - 2xy)\, dx + (y^2 - 2xy)\, dy$ where C is a path from $(-2, 4)$ to $(1, 1)$ along the parabola $y = x^2$.

(10)

2. Let $\alpha : [a, b] \to \mathbb{R}^n$ be a piecewise smooth curve and let $\beta : [c, d] \to \mathbb{R}^n$ be defined by $\beta(t) := \alpha(u(t))$ where $u : [c, d] \to [a, b]$ is continuously differentiable with $u'(t) \neq 0, t \in [c, d]$. Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a continuous vector field. Show that

$$\int_a^b f \cdot \alpha = \pm \int_a^b f \cdot \beta.$$

(10)

3. A region S in $\mathbb{R}^3$ is bounded by the three coordinate planes and the plane $x + 2y + 3z = 6$. Calculate its volume as a multiple integral. (10)

4. Let $u, v : S(r) \to \mathbb{R}$, $S(r) := \{(x, y) : x^2 + y^2 < r\}, r > 1$, be continuously differentiable. Let $f(x, y) := u(x, y)\vec{i} + v(x, y)\vec{j}$ and $g(x, y) := (\partial_x v(x, y) - \partial_y v(x, y))\vec{i} + (\partial_x u(x, y) - \partial_y u(x, y))\vec{j}$. Evaluate $\iint_{S(1)} f \cdot g \; dxdy$. (10)